Grade 8

CONTENT BOOKLET: TARGETED SUPPORT MATHEMATICS

Term 2

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust

(NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District

Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

TOPIC 1: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' which counts for 30% in the final exam.
- The unit covers all the basics of Algebra to lay the groundwork for high school Algebra.
- This topic was started in Term 1 so the learners should already have a good understanding of the basics involved and be ready to move on to the next level covered in Term 2.
- Remember that it is always important to practice and reinforce mental maths wherever possible. No matter what topic is being taught, mental maths across all four operations remains of great importance. As a teacher, you should be incorporating it as often as possible.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae or equations	• Add and subtract like terms in expressions • Multiply integers and monomials by: monomials, binomials and trinomials • Divide monomials, binomials and polynomials by integers or monomials • Simplify algebraic expressions • Find squares, square roots, cubes and cube roots of single algebraic terms • Determine the numerical value of an expression using substitution	• Find the product of two binomials • Square a binomial • Factorise algebraic expressions by finding a common factor, difference of two squares and trinomials • Simplify algebraic fractions by using factorisation

GLOSSARY OF TERMS

Term	Explanation / Diagram
Expression	A mathematical statement which can include variables (letters), constants and operations. Example: 2b + 3c
Product	The answer to a multiplication question.
Sum	The answer to an addition question.
Difference	The answer to a subtraction question.
Quotient	The answer to a division question.
Term	Part of an algebraic expression. Each term is separated by '+' or '–' signs. Example: 3a + 2b has two terms.
Coefficient	A number or symbol multiplied with a variable in a term. Example: 3 is the coefficient of y in 3y 2a is the coefficient of b in the term 2ab
Variable	Letters of the alphabet which could represent different values. Example: In the expression m + 2, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Constant	A number making up a term on its own in an expression. Example: a + 3b – 10: -10 is the constant. Constants cannot change value.
Substitution	Replacing a variable (letter of the alphabet) with a number to perform a calculation. Example: If b = 3 , then b + 2 = 3 + 2 = 5
Monomial	One term expression.
Binomial	Two term expression.
Trinomial	Three term expression.
Polynomial	More than one term expression (two or more).
Like term	Terms that have exactly the same variables. Example: 2a and 4a are like terms and can be added or subtracted 3abc and 10abc are like terms and can be added or subtracted
Unlike term	Terms that do not have exactly the same variables. Example: 3a and 2b are unlike terms and cannot be added or subtracted x and y are unlike terms and cannot be added or subtracted
Exponent	In the example a^2. The '2' (or squared) is the exponent. It is the number or variable written at the top in smaller font to show how many times the base needs to multiplied. For example, $a^2 = a \times a$

SUMMARY OF KEY CONCEPTS

Multiplying monomials by monomials

1. A good understanding of the laws of integers and exponents are both important in this section.
2. Coefficients are multiplied using the integer rules and variables are multiplied using the exponent rules.
3. For example: $(-3x)(2x) = -6x^2$

   [-3 x 2 was done using the laws of multiplying integers and $x \times x$ was done using the laws of exponents]
4. More complicated questions can be asked but the basic rules remain the same.

For example: $(-3a^2b^3)(-2ab^2)(2a^4b) = 12a^7b^6$

Teaching Tip: In order to assist learners in dealing with a problem of this nature, show them how to do it in smaller and easier to deal with pieces by following these steps:

- First decide on the sign in the answer (positive or negative)
- Multiply the integers
- Multiply the variables one at a time

This way they can focus on each piece of the question without letting other bits get in the way. In other words, once the sign of the answer has been decided there is no need to be concerned with all the minus signs – they will have already been used, then it will be easier to just say 3 x 2 x 2. And once the integers and the first variable has been done, they will only need to focus on the $b^3 \times b^2 \times b$.

This method can make these complicated questions more manageable.

$(-3a^2b^3)(-2ab^2)(2a^4b) = 12a^7b^6$

Raising monomials to further powers

1. Raising a monomial to a further power can be done by following these steps. Decide on the sign that the answer will be in. Remember that a negative number raised to an even power will be positive and a negative number raised to an odd power will be negative.
   However, it is also worth noting that the above only applies if the negative is INSIDE the bracket and is being raised to the exponent outside.
   For example: $-x^2$ DOES NOT equal x^2. The negative is not raised to the exponent.
   However: $(-x)^2 = x^2$ because $-x \times -x = +\mathrm{x}^2$
   This needs to be shown to learners.
2. Raise the coefficient to the power using the laws of exponents.
3. Raise each variable to the power using the laws of exponents.

This can include making use of both of the following rules:

a. $(\mathrm{a}^{\mathrm{m}})^{\mathrm{n}} = \mathrm{a}^{\mathrm{m} \times \mathrm{n}} = \mathrm{a}^{\mathrm{mn}}$

b. $(\mathrm{ab})^{\mathrm{n}} = \mathrm{a}^{\mathrm{n}}\mathrm{b}^{\mathrm{n}}$

For example: $(-4a^2)^3$

- Decide on the sign. A negative integer cubed will give a negative answer.
- Raise the coefficient to the power. $(4^3 = 64)$
- Raise the variable to the power using the laws of exponents. $(a^2)^3 = a^6$
- Final answer: $-64\mathrm{a}^6$

Important note: Learners are not expected to write each of these steps in bits, they will just be calculating them one at a time then writing the final solution as one whole answer.

A further example: $-(-2x^3)^2 + 3(2x^2)^3$

$= -(4x^6) + 3(8x^6)$

$= -4x^6 + 24x^6$

$= 20x^6$

Multiplying polynomials by monomials using the distributive law

Brackets show that multiplication needs to take place. If the terms inside the brackets are unlike, the DISTRIBUTIVE LAW is required.

Examples:

1. $3(\mathrm{a}+\mathrm{b})$ (the 3 needs to be multiplied by both terms inside the bracket)
   $=3\mathrm{a}+3\mathrm{b}$

   Integer and exponent rules all apply!

2. $-2(\mathrm{m}-\mathrm{n})$
   $=-2\mathrm{m}+2\mathrm{n}$

3. $3a(a+b)-2(a^2-ab)$
   $=3a^2+3ab-2a^2+2ab$
   $=a^2+5ab$

4. $-x^3(-x+xy)$
   $=x^4-x^4y$

Dividing monomials by monomials

When dividing only one term by another term, the laws of integers and exponents apply.

The following steps can be used to do the division:

- Use the integer rules to divide the coefficients
- Divide each one of the variables using the laws of exponents
  For example:

1. $\frac{10ab}{5a}=2b$ $(10\div5=2\,;\,a\div a=1\,;\,b\div1=b)$
2. $\frac{-8m^5n^2}{2m^2n}=-4m^3n$

Teaching Tip: If learners are struggling with this, encourage them to write it out in full and simplify one 'bit' at a time The following example will be done like this. But remember, this is certainly not essential – it is merely a tool to help learners not managing easily.

3. $\frac{-20x^3y^5}{-4x^2y^3}=\frac{-20.\,x\,.\,x\,.\,x\,.\,y\,.\,y\,.\,y\,.\,y\,.\,y}{-4\,.\,x\,.\,x\,.\,y\,.\,y\,.\,y}=5xy^2$

Dividing polynomials by monomials

When more than one term is divided by only one term, each term in the dividend (numerator) needs to be divided by the divisor (denominator). This then becomes a number of division questions involving the division of a monomial by a monomial.

For example:

1. $\frac{10x^2 - 5x + 5}{x}$

$= \frac{10x^2}{x} - \frac{5x}{x} + \frac{5}{x}$

$= 10x - 5 + \frac{5}{x}$

2. $\frac{4x + 8}{2} + \frac{5x - 3}{5}$

$= \frac{4x}{2} + \frac{8}{2} + \frac{5x}{5} - \frac{3}{5}$

$= 2x + 4 + x - \frac{3}{5}$ $\quad (4 - \frac{3}{5} = \frac{4}{1} - \frac{3}{5} = \frac{20}{5} - \frac{3}{5} = \frac{17}{5})$

$= 3x + \frac{17}{5}$

Determining squares, square roots, cubes and cube roots of algebraic expressions

Learners should already be familiar with squaring, cubing, square rooting and cube rooting. The extra skill needed at this stage is to extend this knowledge to algebraic expressions.

Teaching tip: It is advisable again to encourage learners to find the root (square or cube) of each base in the expression one at a time as opposed to trying to look at the question as one 'big' thing to do and struggling with it.

For example:

$\sqrt{4x^2}$: The focus should be on finding the square root of 4 and then finding the square root of x^2

$(4x^2)^2$: The focus here should be on first squaring the 4 and then squaring the x^2.

$\sqrt{4x^2} = 2x$

$(4x^2)^2 = 16x^4$

If a negative integer is involved, remind learners to focus on that first before moving on to the other bases.

For example: $(-3a^2b^5)^2 = 9a^4b^{10}$

Steps followed to find this solution: First square the negative, then square the 3, then use the exponential laws to square the a^2 and then the b^5

Substitution

Because the letters represent variables and can take on many values we need to be able to substitute these variables for certain values.

Examples:

1. Find $5a + 2b$ *if* $a = 3$ *and* $b = 2$

$$\begin{aligned} 5a + 2b &= 5\,(3) + 2\,(2) \quad \text{(note the use of brackets instead of times signs)} \\ &= 15 + 4 \\ &= 19 \end{aligned}$$

2. Find: $3x - y + 2v$ *if* $x = -1$ $y = -2$ $v = -3$

$$\begin{aligned} 3x - y + 2v &= 3\,(-1) - (-2) + 2(-3) \\ &= -3 + 2 - 6 \\ &= -7 \end{aligned}$$

3. Find: $4x^2 - y^3$ *if* $x = -1$ $y = 2$

$$\begin{aligned} 4x^2 - y^3 &= 4(-1)^2 - (2)^3 \\ &= 4(1) - 8 \\ &= 4 - 8 \\ &= -4 \end{aligned}$$

NOTE: integer and exponent work needs to be very good!

Teaching tip: Experience has shown that learners not only find substitution easier if they follow the steps shown below but also make much less errors:

- Rewrite the expression given, but every time a variable appears, open and close a bracket
- Now check what value has been allocated to each variable and put it inside the empty bracket in the correct variables place.
- Use BODMAS to find the value of the expression

For example:

If $p = 2,\ q = -2\ and\ \mathrm{r} = 3$ find the value of $2p^2q + (qr)^2$

STEP 1: $2(\ \ )^2(\ \ ) + ((\ \ )(\ \ ))^2$

STEP 2: $2(2)^2(-2) + ((-2)(3))^2$

STEP 3: $2(4)(-2) + (-6)^2$

$= -16 + 36$

$= 20$

(NOTE: Step 1 and 2 would not show as 2 steps when the learners have completed the question, as step 2 would have been done as soon as step 1 was complete)

TOPIC 2: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area ‘Patterns, Functions and Algebra’ which counts for 30% in the final exam.
- The unit covers the solving of basic Equations.
- It is important to note that learners must master the skills of solving simple Equations. This topic was started in Term 1 so the learners should already have a good understanding of the basics involved and be ready to move on to the next level covered in Term 2. Mastering these basic equations is essential - Grade 9 to 11 have more difficult equations and the learners will not manage these if the basics are not fully understood.
- Remember that it is always important to practice and reinforce mental maths wherever possible. No matter what topic is being taught, mental maths across all four operations remains of great importance. As a teacher, you should be incorporating it as often as possible.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Write number sentences to describe problem situations • Solve and complete number sentences by inspection and trial & improvement • Determine the numerical value of an expression by substitution • Identify variables and constants	• Use additive and multiplicative inverses to solve equations • Use the laws of exponents to solve equations • Use substitution in equations to generate tables of ordered pairs	• Use factorisation to solve equations • Solve equations of the form: a product of factors = 0

GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	A mathematical statement with an equal sign that includes a variable. Example: $3x - 5 = 20$
Expression	An algebraic statement consisting of terms with variables and constants. There is no equal sign. Example: $3a + 2b$
Formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value. Example: Area $= l \times b$ This formula finds area of a rectangle and only measurements can replace the $l \text{ and } b$
Variable	Letters of the alphabet which could represent different values. Example: In the expression $m + 2$, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Like Terms	Terms that have exactly the same variables. Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and $10abc$ are like terms and can be added or subtracted
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and division are the inverse operation of each other.
Substitution	Replacing the variable with a number. It can be used in Equations to check if the answer is correct by checking if the Left Hand Side (LHS) is equal to the Right Hand Side (RHS) of the original equation.

SUMMARY OF KEY CONCEPTS

Solving equations using additive and multiplicative inverses

In order to do this, a learner needs to:

1. Understand BODMAS and how to 'undo' operations (as was done in the Functions and Relationships section while looking for input value when you had the output value)
2. Be able to collect like terms (as was done in the Algebra section)

The most important rule to keep in mind when dealing with equations:

Whatever you do to one side of an equation you must ALWAYS do to the other side in order to keep the equation balanced

For example:

1. $\frac{4x}{5} - 6 = -2$

$\frac{4x}{5} - 6 + 6 = -2 + 6$

$\frac{4x}{5} = 4$

$5 \times \frac{4x}{5} = 4 \times 5$

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

'undo' order of operations:

- + 6 to both sides
- x 5 both sides
- ÷ 4 both sides

$3(x + 2) - 2x = 10$

$3x + 6 - 2x = 10$

$x + 6 = 10$

$x + 6 - 6 = 10 - 6$

$x = 4$

- remove brackets using distributive law
- collect any like terms
- use inverse operations to get the variable on its own

Solving equations by using the laws of exponents

Consider the following statement:

$5^6 = 5^x$

It should be easy to see that $x = 6$. This is because the bases are the same.

If you ever need to solve for a variable in the exponent position, it is always necessary to get the bases to be equal on each side.

Examples:

1. $4^x = 64$

   $4^x = 4^3$

   $\therefore x = 3$

2. $2.2^x = 16$

   $2.2^x = 2^4$

   $2^{1+x} = 2^4$

   $\therefore 1 + x = 4$

   $x = 3$

- change all bases to 2
- use rules of exponents to simplify
- once bases are the same, make the exponents the same and solve

Setting up equations to describe given situations

1. This section is important in order to be able to solve word problems.

In order to be able to do this, it is important that learners have a good understanding of how to change 'words' into 'maths'.

Add (+)	Subtract (-)	Multiply (x)	Divide (÷)	Equals/Inequality (=,<,>,≤,≥)
add plus more than in addition to also all together in all	less than minus	each all together in all double (x2) triple (x3)	each groups per half (÷2)	is more than less than greater than fewer than

The majority of textbooks have a good summary sheet of this type of table. If your school does not have access to a textbook there is a table that can be used on the resource page.

A common phrase used in word problems is that of 'consecutive integers'. This is covered in the table provided.

The difference between an expression, a formula and an equation.

Ensure you know the difference between an equation, an expression and a formula.

An **algebraic expression** has terms, variables and constants but no equal signs.

For example: $2x^2 - 3x + 5$

A **formula** is similar to an expression in that it has terms and variables and also similar to an equation in that it has an equal sign but the difference is that the variables represent specific types of quantities (eg measurements or interest rates etc)

For example: $V = lbh$

(Volume = length x breadth x height. Each variable represents a specific measurement in this case)

An **equation** is an algebraic statement with an equal sign. The variable represents one specific value that will make the equation true. To find this value the equation needs to be solved.

For example: $2a - 5 = 8$

Analysing and interpreting equations

To analyse an equation means to know what each variable and constant will represent.

To interpret an equation means to know what the equation means (represents) and to find the solution and understand what has been found.

For example: A car hire company charges a once off fee of R200 per car then charges R75 per day thereafter. An equation to find the cost of hiring a car could be stated as follows:

$C = 200 + 75x$

In this case, x represents the number of days the car will be hired for – it is the value that can vary and is therefore the variable in the equation. The other variable is the 'C' which represents the total cost for hiring the car and depends on the number of days it will be hired for.

This formula can now be used to answer questions on the car hire.

If a man is to hire a car for 5 days, what will it cost him?

$$
\begin{aligned}
C &= 200 + 75\,(5) \\
&= 200 + 375 \\
&= 575
\end{aligned}
$$

Notice that this question required a knowledge of substitution.

If it cost R800 to hire a car, how many days was the car hired for?

$$
\begin{aligned}
800 &= 200 + 75x \\
800 - 200 &= 200 + 75x - 200 \\
600 &= 75x \\
\frac{600}{75} &= \frac{75x}{75} \\
8 &= x
\end{aligned}
$$

Notice that this question required a knowledge of solving equations.

RESOURCES

Phrase	Expression
the sum of nine and eight	$9+8$
the sum of nine and a number x	$9+x$
the sum of a and 8 is equal to 12	$a+8=12$
nine increased by a number x	$9+x$
fourteen decreased by a number p	$14-p$
seven less than a number t	$t-7$
the product of 9 and a number n	$9n$
thirty-two divided by a number y	$\frac{32}{y}$
five more than twice a number	$2n+5$
the product of a number and 6	$6n$
is divided by y	$\frac{x}{y}$
seven divided by twice a number	$\frac{7}{2n}$
three times a number decreased by 11	$3n-11$
the value of 'd' is twice that of 'e'	$d=2e$
f is half of g	$f=\frac{1}{2}g$
m is a quarter of n	$m=\frac{1}{4}n$
Consecutive numbers are numbers that come one after the other.	
Three consecutive natural numbers can be represented as:	$x+(x+1)+(x+2)$
Three consecutive even numbers can be represented as follows:	$x+(x+2)+(x+4)$
Reason: Even numbers go up in twos	
Three consecutive odd numbers can be represented as follows:	$x+(x+2)+(x+4)$
This can be confusing as it looks the same as the even numbers. But odd numbers also go up in twos! If the situation is represented correctly according to the question an odd number will be gained in the answer.	

TOPIC 3: CONSTRUCTION OF GEOMETRIC FIGURES

INTRODUCTION

- This unit runs for 8 hours.
- It is part of the Content Area 'Space and Shape' which counts for 25% in the final exam.
- The unit covers geometric constructions.
- Learners must be encouraged to investigate properties of triangles and quadrilaterals while constructing. The focus should be on the sum of the interior angles as well as the lengths of the opposite sides.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Measuring angles accurately with a protractor • Classify acute, obtuse and reflex angles • Accurately construct angles, circles, parallel lines and perpendicular lines	Accurately construct (without a protractor): • bisecting lines and angles • perpendicular lines from a given point • triangles • quadrilaterals By construction investigate: • The sum of the interior angles of a triangle • The size of angles in an equilateral triangle • The sides and base angles of an isosceles triangle • the sum of the interior angles of quadrilaterals • the sides and opposite angles of parallelograms	By construction investigate: • The diagonals of rectangles, squares, parallelograms, rhombi and kites • The sum of interior angles of polygons • The minimum conditions for two triangles to be congruent

GLOSSARY OF TERMS

Term	Explanation / Diagram
Lines	Lines exist within a plane and continue to both sides forever and will have an arrow on both ends. Think of the number lines.
Line segment	A type of line that extends from one point to another point
Ray	A type of line that starts at a certain point and ends with an arrow as it can continue forever
Protractor	A mathematical instrument that is used to draw or measure angles. The mathematical instrument can either be round or half a circle.
Point	A dot representing an exact spot on a plane. It can be seen as the starting point of a ray.
Obtuse angle	An angle that is greater than 90° but smaller than 180°
Acute angle	An angle that is smaller than 90°
Reflex angle	An angle that is greater than 180° and smaller than 360°
Straight lines	An angle of exactly 180°
Right angle	An angle of exactly 90°
Revolution	An angle of exactly 360°
Horizontal Line	A line that is drawn from left to right
Vertical Line	A line that is drawn from the top to the bottom.
Perpendicular Line	This line would meet another line making a 90° angle
Bisect	This means to cut a line or an angle in half
Scalene triangle	A type of triangle that does not have any sides equal to each other and no angles would have the same measured size.
Isosceles triangle	A triangle where two sides are equal and the two base angles to these equal sides are also equal.
Equilateral triangle	A triangle that has all three sides equal and all three interior angles are 60°
Quadrilaterals	A shape with four sides
Rectangle	A quadrilateral with four right angles and two pairs of parallel sides. The opposite sides are equal in length.
Square	A type of rectangle but all four sides are equal in length.
Parallelogram	A quadrilateral with two pairs of opposite sides equal and parallel. Two opposite angles are acute and two are obtuse.
Rhombus	A type of parallelogram where all four sides are equal in length.
Kite	A 4 sided figure with adjacent sides equal in length.
Adjacent	Next to
Arc	Part of a circumference of a circle. Made with a compass.

SUMMARY OF KEY CONCEPTS

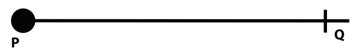
Construction of perpendicular lines

Constructing a perpendicular bisector:

For example: Construct a perpendicular bisector of line PQ with a length of 10cm

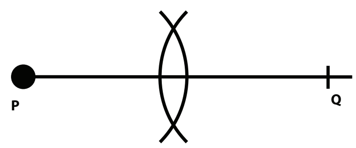
Step 1:

- Draw a line longer than 10cm.
- PQ = 10cm and mark it off on the longer line drawn
- Label the points P and Q



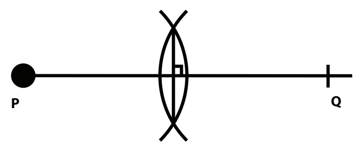
Step 2:

- Place your compass on P and draw an arc more than halfway (more than 5cm in this case) above AND below the line
- Repeat the process from Q, keeping your compass the same length, making sure the arcs cross each other.



Step 3:

- Using your ruler, draw a line joining the two places where the arcs cross

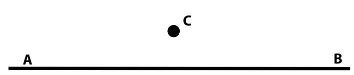


Constructing a perpendicular line from a point above a given line

For example: Construct a line perpendicular to a line AB from point C

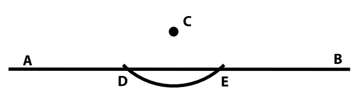
Step 1:

- Draw a line AB and mark a point C anywhere above it



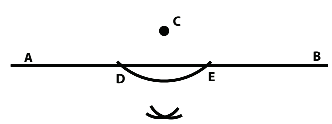
Step 2:

- Place your compass on point C and draw an arc making sure it intersects the line AB in two places. Name these points of intersection D and E



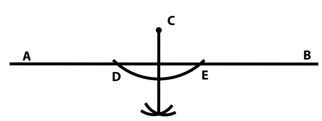
Step 3:

- Place your compass on D and draw an arc below the line AB
- Keeping your compass the same length, repeat the process from E making sure you cross the first arc.



Step 4:

- Draw a line from point C through where the two arcs meet.

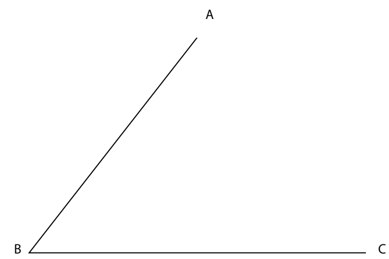


Construction of angles

Bisecting an angle

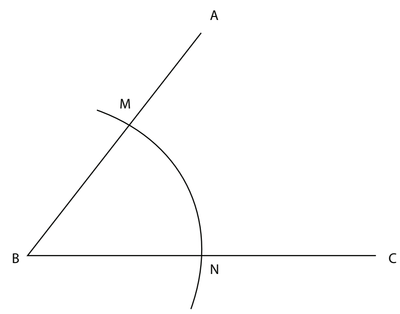
Step 1:

- Draw any size angle ABC



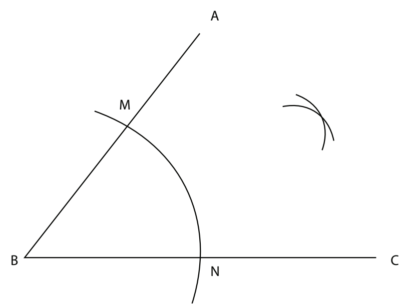
Step 2:

- Place your compass on B and draw an arc crossing both line segments. Call these points M and N



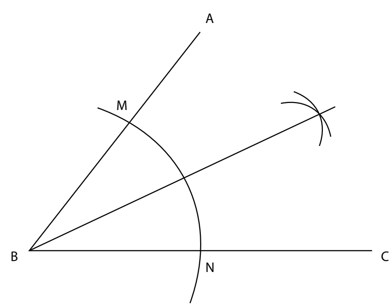
Step 3:

- Place your compass on point M, draw an arc inside the angle
- Keeping your compass the same length, place your compass on N and intersect the arc made from M



Step 4:

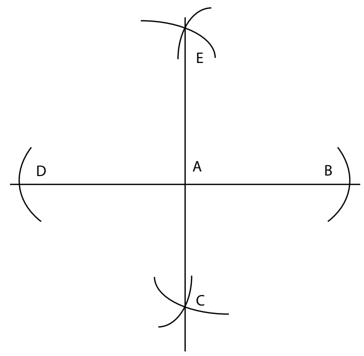
- Using your ruler, draw a line from B through the point of intersection of the two arcs.



Constructions of a right angle and a 45° angle using a compass

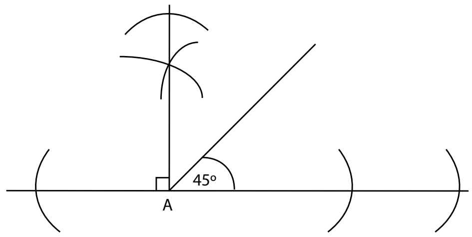
Step 1:

- Follow the instructions on 'Constructing a perpendicular line from a point above a given line'
- This will give a 90^0 angle



Step 2:

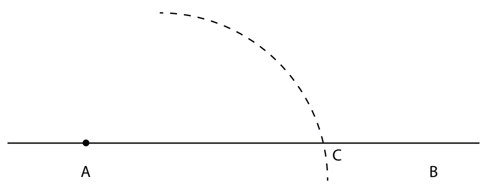
- Follow the instructions on 'Bisecting an angle'
- This will give a $45°$



Construction of 60° angles

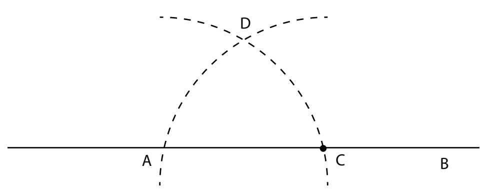
Step 1:

- Draw a line AB of any length
- Place your compass on A, and draw a long arc to intersect AB. Call this point C



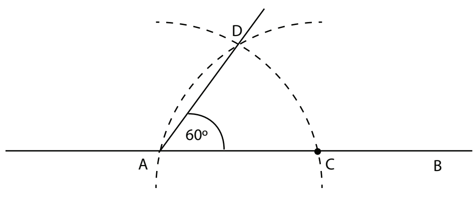
Step 2:

- Keeping your compass the same length, place your compass on C and make an arc across the first arc.
- Call this point D



Step 3:

- Join A to D
- This is a 60° angle



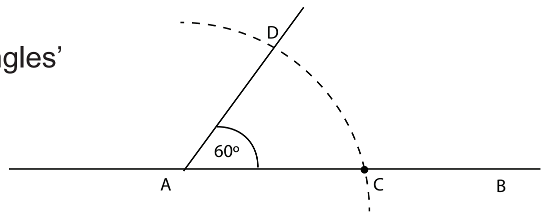
Constructions of a 30° angle using a compass

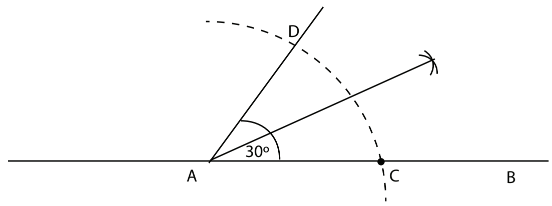
Step 1:

- Follow the instructions 'Construction of 60° angles'

Step 2:

- Follow the instructions ''Bisecting an angle'
- This will give a 30o angle

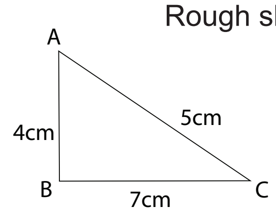




Construction of triangles

ALWAYS draw a rough sketch first, labelling the triangle correctly according to the information given

Rough sketch



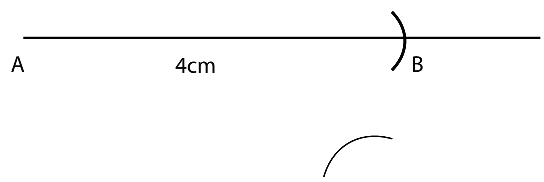
Construction of a Scalene triangle

For example: Construct ∆ABC,

with AB = 4cm; AC = 5cm and BC = 7cm

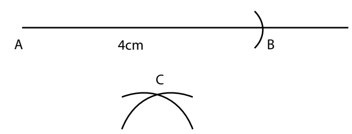
Step 1:

- Draw a line more than 4cm
- measure 4cm accurately using your compass on your ruler
- mark off the exact measurement on the line you drew, marking the points A and B



Step 2:

- Set your compass to 5cm, place your compass on B and make an arc.

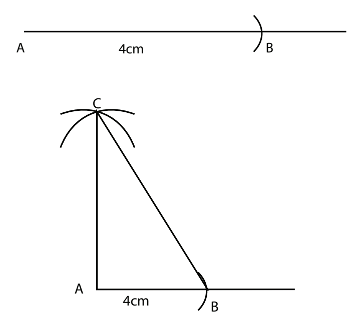


Step 3:

- Set your compass to 7cm, place your compass on A and make an arc. Ensure it crosses the arc from step 2. Call this point C

Step 4:

- Join BC and AC



Construction of an Equilateral triangle

For example: Construct $\triangle PQR$, with PQ = PR = QR = 6cm

Step 1:

- Follow the steps of ‘Construction of a Scalene triangle’ but keep your compass measured at 6cm all the time.

Construction of an Isosceles triangle

For example: Construct $\triangle DEF$, with DE = DF = 8cm

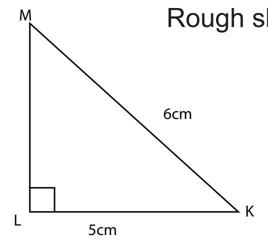
Step 1:

- Follow the steps of ‘Construction of a Scalene triangle’ using 8cm for the first two measurements then join the final sides.

Construction of a Right-angled triangle

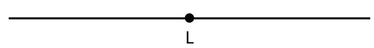
For example: Construct $\triangle KLM$, with $\hat{L} = 90°$, and KL = 5cm and KM = 6cm.

Rough sketch



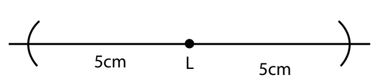
Step 1:

- Draw a horizontal line more than twice the length of the shorter side given.
- Place the point that needs to be the right angle near the centre of the line (in this case ‘L’)



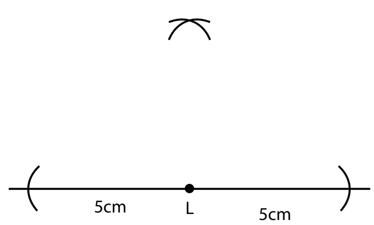
Step 2:

- Measure the length of the shorter side with the compass and a ruler.
- Place the compass point on the marking for the right angle and mark the length off with an arc on either side (to the left and right) of the point on the horizontal line



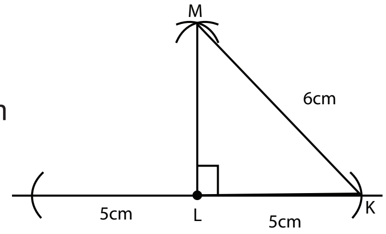
Step 3:

- Measure the length of the hypotenuse using the compass and a ruler.
- Place the compass point on the first marked arc from the previous step and make a new arc in the space above the marked point for the right angle.
- Move the compass point to the other arc made in the previous step and make an arc to cross the first arc just made.



Step 4:

- Use the crossing of the arcs to make the right angle (at 'L' in this case) and also to join up with the other point to form the hypotenuse and hence the triangle



Construction of parallel lines

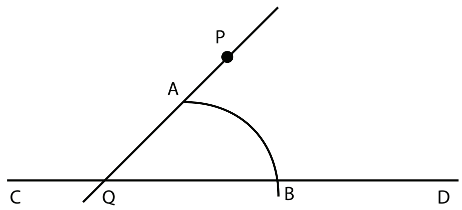
Constructing a line parallel to one already given

Example: Construct a line parallel to CD through the point P

Step 1:

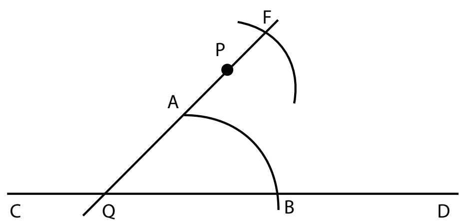
Place your compass on Q (the point of intersection of the 2 lines given) and

draw an arc that will intersect both lines. Call these points A and B



Step 2:

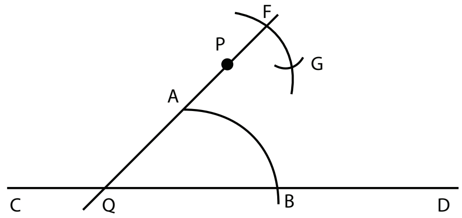
Keeping your compass the same length, place your compass on point P and draw an arc (it will only cross one line). Call this point F



Step 3:

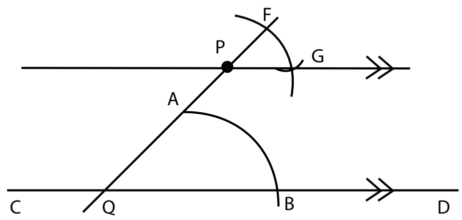
Using your compass, measure out the length of the arc that touches both lines (the one drawn in step 1 - AB) Keeping your compass the same length, place your compass on F and

mark off this length on the arc. Call this point G



Step 4:

Using your ruler, draw a line through PG

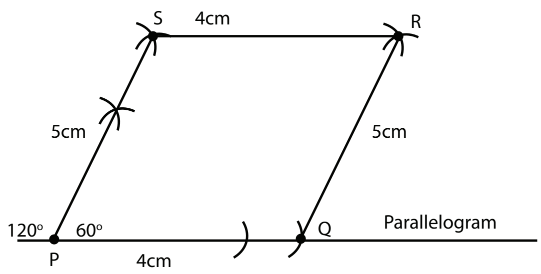


Construction of quadrilaterals

Construction of a Parallelogram

Remember: A parallelogram is a 4 sided figure with both pairs of opposite sides equal and parallel to each other.

Example: Construct a parallelogram PQRS, with $\hat{P} = 60°$, PQ = 4cm and PS = 5cm



Step 1:

- Draw a rough sketch

Step 2:

- Construct a 60° at P

Step 3:

- Construct a 4cm line from P to Q

Step 4:

- Construct a 5cm line from P to S

Step 5:

- Construct a line through S parallel to PQ

Step 6:

- Construct a line through R parallel to PS

Construction of a Rhombus

Remember: A rhombus is a parallelogram with all 4 sides equal in length so the steps are the same as constructing a parallelogram.

RESOURCES

The following link has excellent instructions with regards to constructions if access to the internet is possible. Each construction is shown on a board where learners can see what the compass is 'doing'.

http://www.mathopenref.com/constructions.html

TOPIC 4: GEOMETRY OF 2D SHAPES

INTRODUCTION

- This unit runs for 8 hours
- It is part of the Content Area ‘Space and Shape’ which counts for 25% in the final exam.
- The unit covers Triangles, Quadrilaterals, Congruent and Similar shapes.
- Remember that it is always important to practice and reinforce mental maths wherever possible. No matter what topic is being taught, mental maths across all four operations remains of great importance. As a teacher, you should be incorporating it as often as possible.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describing, sort, name and compare triangles to their sides and angles, focusing on equilateral, isosceles and right-angled triangles. • Describing, sort, name and compare quadrilaterals in terms of the lengths of their sides, parallel and perpendicular sides and sizes of angles	Identify and write clear definitions of: • Equilateral triangles • Isosceles triangles • Right-angled triangles Identify and write clear definitions of: • Parallelogram • Rectangle • Rhombus • Square • Trapezium • kite • congruent shapes • similar shapes Identify and describe the properties of: • congruent shapes • similar shapes Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals.	• Revision of all previous work concerning triangles and quadrilaterals

GLOSSARY OF TERMS

Term	Explanation / Diagram
Triangle	A 3 sided closed shape
Square	A quadrilateral with all angles 90° and all sides equal
Circle	A round plane figure whose boundary (the circumference) consists of points equidistant from a fixed point (the centre).
Quadrilateral	A 4 sided closed shape
Parallelogram	A quadrilateral with two pairs of parallel sides
Rectangle	A quadrilateral .with all angles being 90° and opposite sides equal and parallel
Rhombus	A quadrilateral with two pairs of parallel sides and all 4 sides equal
Trapezium	A quadrilateral with one pair of parallel sides
Kite	A quadrilateral with two pairs of adjacent sides equal
Equilateral triangle	A triangle with all 3 sides and all 3 angles equal
Isosceles triangle	A triangle with 2 equal sides
Scalene triangle	A triangle with no equal sides
Right-angled triangle	A triangle with one right angle
Acute-angled triangle	A triangle with 3 acute angles
Obtuse-angled triangle	A triangle with one obtuse angle
Parallel lines/sides	Lines exactly the same distance apart at all points. Has the same slope.
Corresponding	In the same position
Congruent	Exactly the same. Identical. Equal sides and equal angles
Similar	Looks the same. Equal angles and sides in proportion

SUMMARY OF KEY CONCEPTS

Classifying Triangles

Triangles are named using their sides and the size of angles

1. According to sides:
   Scalene – all three sides are different in length
   Isosceles – two of the sides are equal in length
   Equilateral – all three sides are equal in length

2. According to angles:
   Right-angled – one of the angles is a right angle
   Acute-angled – all three angles are acute
   Obtuse-angled – one of the angles is an obtuse angle

In general each triangle is named in both ways.

For example: A right-angled scalene triangle.

Classifying Quadrilaterals

1. Quadrilaterals are 4-sided shapes
2. There are 6 basic quadrilaterals dealt with, namely:
   a. Parallelogram
   b. Square
   c. Rectangle
   d. Rhombus
   e. Trapezium
   f. Kite

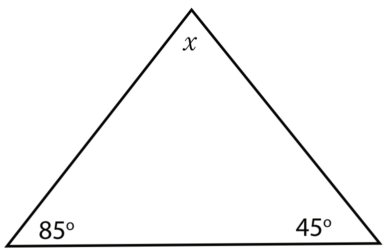
3. The properties of each of these quadrilaterals is summarised in the table below:

Property:	Parallelogram	Square	Rectangle	Rhombus	Trapezium	Kite
All sides are equal		X		X		
Opposite sides are equal	X	X	X	X		
At least one pair of adjacent sides are equal		X		X		X
All four angles are right angles		X	X			
Both pairs of opposite sides are parallel	X	X	X	X		
Only one pair of opposite sides are parallel					X	

Rules (theorems) regarding triangles

1. The sum of the interior angles of a triangle is 180°

Example:

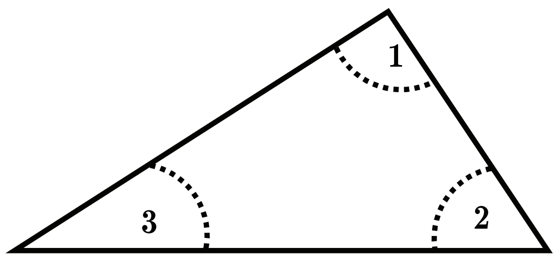


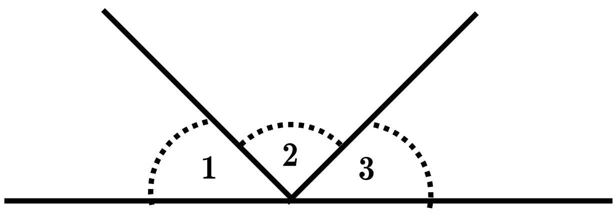
Find x:

STATEMENT	REASON
$x + 85^\circ + 45^\circ = 180^\circ$	$<'s\ of\ \Delta = 180^\circ$
$x + 130^\circ = 180^\circ$	
$x = 50^\circ$	

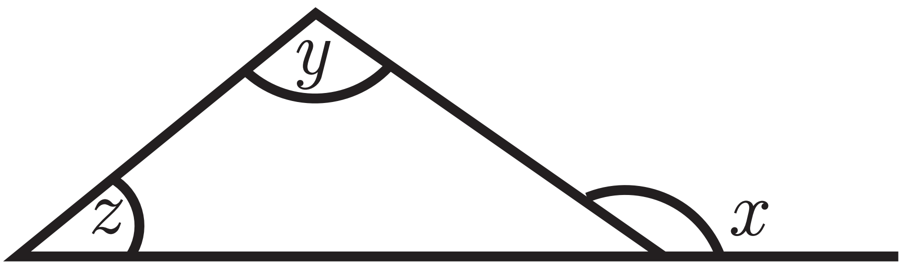
Teaching Tip: To help learners to learn this theorem in a more concrete way try the following:

- Ask learners to draw a triangle and number the angles 1, 2 and 3
- Let them tear off each of these angles
- Ask them to draw a straight line and place each of the three numbered angles on the straight line.
- Learners should now see that the three angles form the straight line. Knowing that a straight line is 180^o should now show learners that the angles of a triangle will also add up to 180^o.



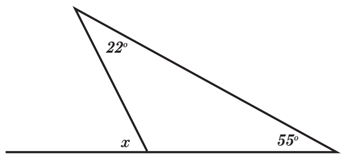
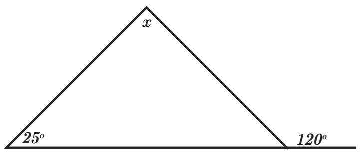


2. The exterior angle of a triangle is equal to the sum of the opposite interior angles



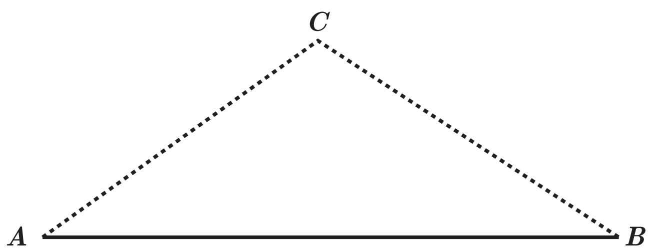
$$x = y + z$$

Examples:

	STATEMENT	REASON
	$x = 22^\circ + 55^\circ$ $x = 77^\circ$	$Ext < of \ \square = sum\ int\ opp\ <'s$
	$x + 25^\circ = 120^\circ$ $x = 95^\circ$	$Ext < of \ \square = sum\ int\ opp\ <'s$

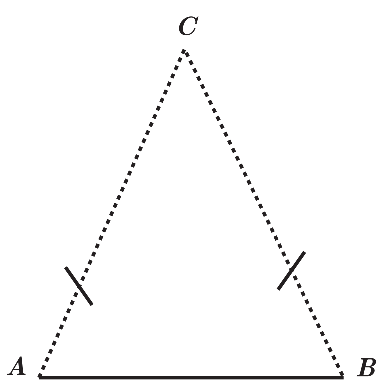
Before we look at the wording of the next theorem it is important for you to understand a word that comes up in geometry quite often.

If we form a triangle ABC from a line segment, it would look like this:



We say that AB **subtends** $\hat{C}$ or $\hat{C}$ is **subtended** from AB

3. In an Isosceles triangle, the angles subtended from the two equal sides are equal.

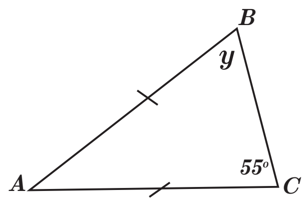
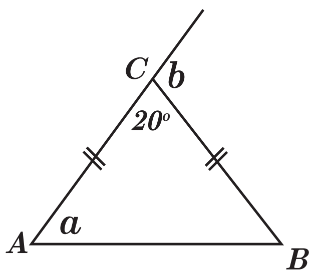


if AC = BC

Then $\hat{A} = \hat{B}$

Examples

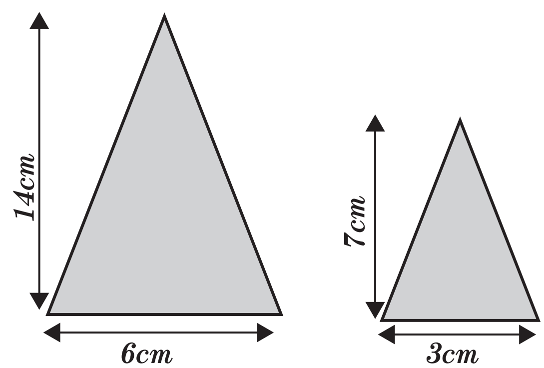
Find the value of the angles marked with a variable

	STATEMENT	REASON
	$y = 55^0$	AB = BC; Equal angles opposite equal sides
	$A\hat{B}C = a$	Equal angles opposite equal sides
	$a + a + 20^0 = 180^0$	$<'s\ of\ □ = 180^0$
	$2a + 20^0 = 180$	
	$2a = 160^0$	
	$a = 80^0$	
	$b = 80^0 + 80^0$	$Ext < of\ □ = sum\ int\ opp\ <'s$
	$b = 160^0$	

Similar and Congruent 2D shapes

1. Similar means the shape looks the same. They are NOT the same size.
   All angles must be equal for a shape to be similar and all sides must be in proportion.
   (It helps to see if the smaller shape will fit perfectly inside the larger shape)
   For example:

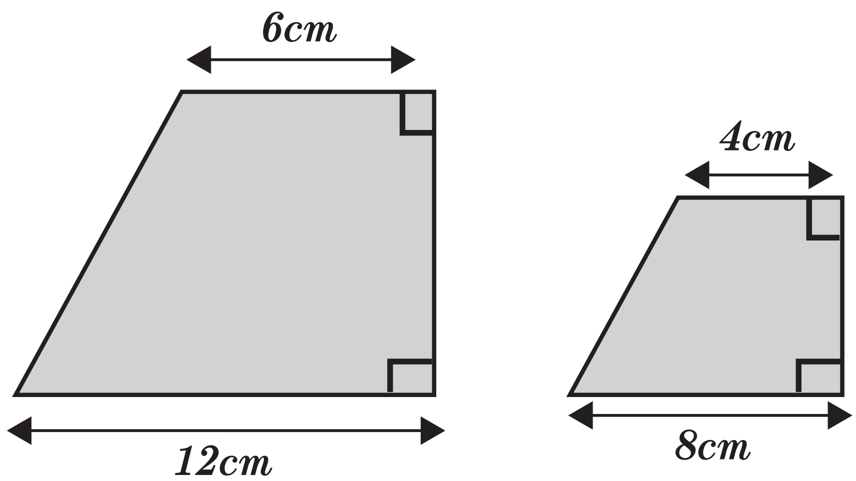
   a.



Note that the sides of this triangle are in the ratio 14:7 and 6:3

Both of these simplify to a ratio of 2:1

b.



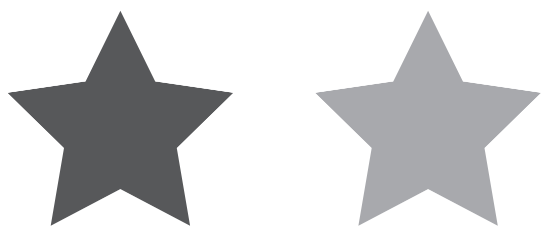
Note that the sides of this trapezium are in the ratio 6:4 and 12:8

Both of these simplify to a ratio of 3:2

2. Congruent means EXACTLY the same. All angles and all sides of one shape are equal to all the angles and sides of another shape.

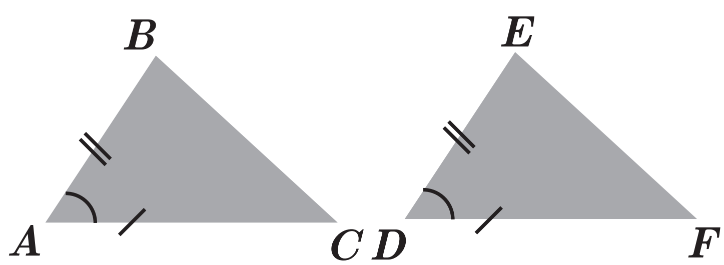
For example:

a.



they are the same size and shape

b.



This work is covered in more detail in Grade 9. It is sufficient now for the learners to understand the idea and meaning behind the words.

TOPIC 5: GEOMETRY OF STRAIGHT LINES

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Space and Shape' which counts for 25% in the final exam.
- The unit covers relationships of angles.
- It is important to note that geometry is all around us and an important area of mathematics to learn. Most people think in terms of shapes and sizes, and understanding geometry helps improve reasoning in this area.
- Remember that it is always important to practice and reinforce mental maths wherever possible. No matter what topic is being taught, mental maths across all four operations remains of great importance. As a teacher, you should be incorporating it as often as possible.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
Define a: • Line segment • Ray • Straight line • Parallel lines • Perpendicular lines	Recognise and describe pairs of angles formed by: • Perpendicular lines • Intersecting lines • Parallel lines cut by a transversal • Solving geometric problems using the relationships between pairs of angles	Recognise and describe pairs of angles formed by: • Perpendicular lines • Intersecting lines • Parallel lines cut by a transversal • Solving geometric problems using the relationships between pairs of angles

GLOSSARY OF TERMS

Term	Explanation / Diagram
Acute angle	An angle between 0° and 90°
Right angle	A 90° angle
Obtuse angle	An angle between 90° and 180°
Reflex angle	An angle between 180° and 360°
Revolution	A 360° angle
Adjacent	Next to
Complementary	Angles that add up to 90°
Supplementary	Angles that add up to 180°
Parallel Lines	Lines exactly the same distance apart at all points. Has the same slope.
Corresponding Angles	Angles that sit in the same position
Alternate Angles	Angles that lie on different parallel lines and on opposite sides of the transversal
Co-interior Angles	Angles that lie on different parallel lines and on the same side of the transversal
Transversal	A line that cuts across a set of lines (usually parallel)

SUMMARY OF KEY CONCEPTS

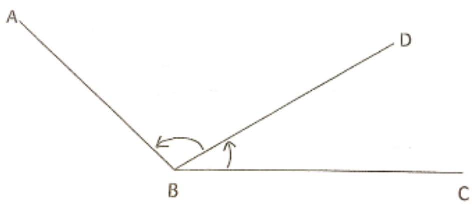
Naming of Angles

Adjacent Angles

Angles are adjacent if they:

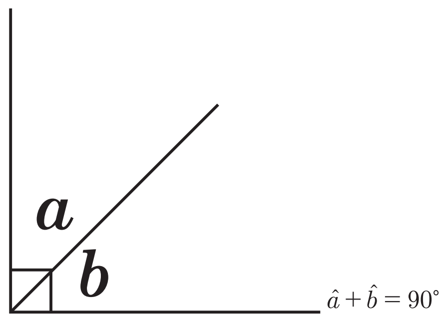
- have a common vertex (in this case, B)
- have a common arm (in this case BD)
- lie on opposite sides of the common arm (one angle on each side of BD)

For angles to be adjacent, all three of the above must be evident.



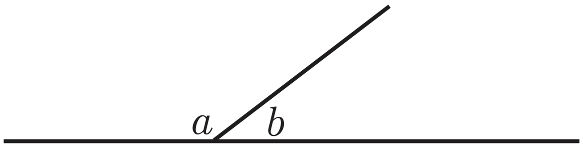
Complementary Angles

Angles that add up to 90°



Supplementary Angles

Angles that add up to 180°



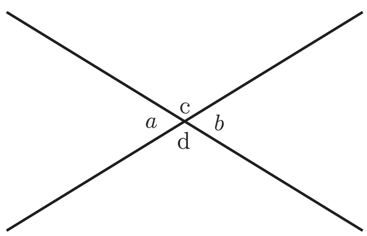
$$a + b = 180^o$$

Teaching Tip: If learners are having trouble remembering these two words (especially in a test situation when they can't ask), suggest the following to them: Write down a 'C' and an 'S'. Draw a line through each.
They should see that one looks more like a '9' (C – Complementary - 90^0) and the other looks more like an '8' (S – Supplementary - 180^0)

$$\begin{array}{cc} \cancel{C} & \cancel{S} \\ \downarrow & \downarrow \\ 9 & 8 \\ (90^o) & (180^o) \end{array}$$

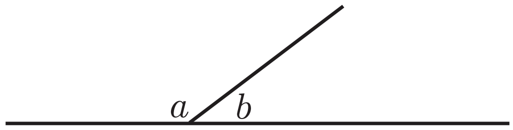
Rules (theorems) involving angles and straight lines

Vertically opposite angles are equal



Statement	Reason
$a = b$	Vert opp <'s equal
$c = d$	Vert opp <'s equal

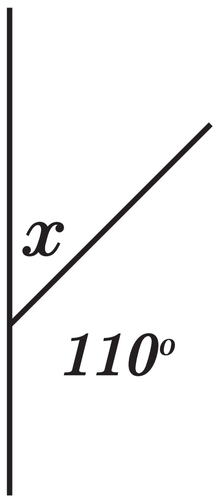
Adjacent angles on a straight line add up to 180°



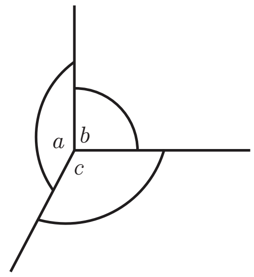
$$a + b = 180^o$$

For example:

Find x

	STATEMENT	REASON
	$x + 110^\circ = 180^\circ$ $x = 70^\circ$	*Adj <'s str line* $= 180^o$

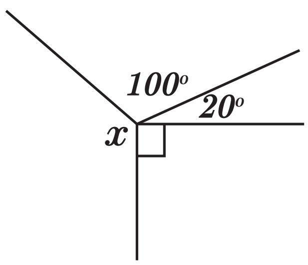
Angles around a point add up to 360°



$a + b + c = 360^o$

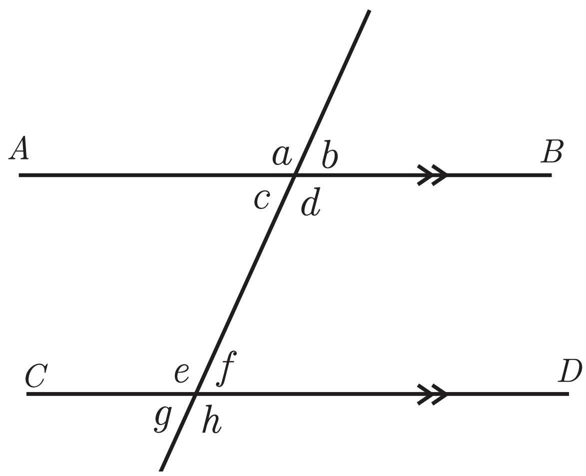
For example:

Find the value of x.

<table>
<tr><th></th><th>STATEMENT</th><th>REASON</th></tr>
<tr><td></td><td>$x + 90^\circ + 20^\circ + 100^\circ = 360^\circ$
$x + 210^\circ = 360^\circ$
$x = 150^\circ$</td><td><’s around a pt = 360°</td></tr>
</table>

Parallel Lines

If two lines are parallel with a transversal (a line crossing through both lines that are parallel to each other) running through them, 8 angles are formed.



Teaching Tip: Ask learners to draw their own pair of parallel lines with a transversal running through them. Using a protractor they should now measure all eight angles formed. Once they have worked by themselves let them discuss with each other what they have found and if they are able to make any conclusions regarding any relationships between any of the angles and their sizes.

Once the learners have investigated for themselves you can show them the following information:

Corresponding angles are equal (these are angles that 'sit' in the same position on each parallel line).

$$b = f \ \ and \ \ a = e \ \ and \ \ c = g \ \ and \ \ d = h$$

Alternate angles are equal (angles that are opposite the transversal to each other and inside the parallel lines)

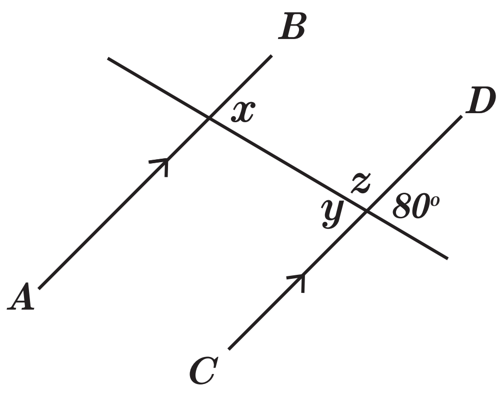
$$c = f \ \ and \ \ d = e$$

1. Co-interior angles are supplementary (add up to 180^o) (angles that are on the same side of the transversal and inside the parallel lines)

$$c + e = 180^\circ \ \ and \ \ d + f = 180^\circ$$

For example:

Find x,y and z

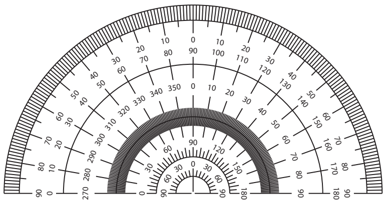


STATEMENT	REASON
$x = 80^{\circ}$	AB//CD; corres <'s equal
$y = 80^{\circ}$	AB//CD; alt <'s equal
$z + 80^{\circ} = 180^{\circ}$	AB//CD; co-int <'s = 180°
$z = 100^{\circ}$	

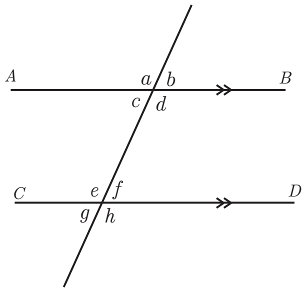
INVESTIGATION:

PARALLEL LINES

You will need a protractor to complete this investigation



If two lines are parallel with a transversal running through them, 8 angles are formed.



1. Measure all of the angles:

a		e	
b		f	
c		g	
d		h	

2. What do you notice?

3. If you had been given the size of ‘a’, what theorems could you have used to find b, c and d?
